Geometry
Chapter 2

Reasoning and Proof

Geometry 2

- This Slideshow was developed to accompany the textbook
  - *Larson Geometry*
  - *By Larson, R., Boswell, L., Kanold, T. D., & Stiff, L.*
  - *2011 Holt McDougal*
- Some examples and diagrams are taken from the textbook.

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2.1 Use Inductive Reasoning

- Geometry, and much of math and science, was developed by people recognizing patterns
- We are going to use patterns to make predictions this lesson

2.1 Use Inductive Reasoning

Conjecture

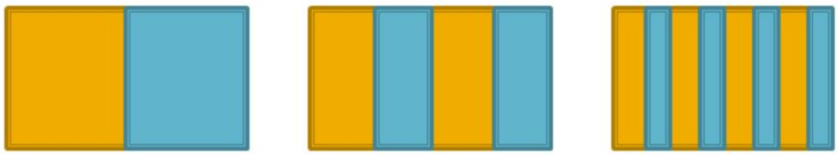
- Unproven statement based on observation

Inductive Reasoning

- First find a pattern in specific cases
- Second write a conjecture for the general case

2.1 Use Inductive Reasoning

- Sketch the fourth figure in the pattern



- Describe the pattern in the numbers 1000, 500, 250, 125, … and write the next three numbers in the pattern

Each number is ½ the previous number: 62.5, 31.25, 15.625

2.1 Use Inductive Reasoning

- Given the pattern of triangles below, make a conjecture about the number of segments in a similar diagram with 5 triangles

- Make and test a conjecture about the product of any two odd numbers

Each figure has two more segments
Third figure has seven segment, so 5th has 7 + 2 + 2 = 11

Product means multiply
Try several: 3(5) = 15; 7(11) = 77; 9(3) = 27
Looks like the product of two odd numbers is **odd**

2.1 Use Inductive Reasoning

- The only way to show that a conjecture is true is to show **all** cases

- To show a conjecture is false is to show **one** case where it is false
  - This case is called a **counterexample**

2.1 Use Inductive Reasoning

- Find a counterexample to show that the following conjecture is false
  - The value of x^2 is always greater than the value of x
- *75 #5, 6-18 even, 22-28 even, 32, 34, 38-46 even, 47-49 all = 22 total*

Sample answer: let x = ½; x^2 = ¼

2.1 Answers and Quiz

- 2.1 Answers
- 2.1 Homework Quiz

2.2 Analyze Conditional Statements

Conditional Statement

- Logical statement with two parts
  - Hypothesis
  - Conclusion
- Often written in If-Then form
  - If part contains hypothesis
  - Then part contains conclusion

If we confess our sins, then He is faithful and just to forgive us our sins. 1 John 1:9

Red is hypothesis, Gray is conclusion

2.2 Analyze Conditional Statements

If-then statements

$p \rightarrow q$

- The if part implies that the then part will happen.
- The then part does NOT imply that the first part happened.

Focus: If you are hungry, then you should eat.
John is hungry, so… (good reasoning)
Megan should eat, so… (not good reasoning)

- ANS: hypothesis: it is Wednesday; conclusion: there is no rec.

2.2 Analyze Conditional Statements

Converse

$q \rightarrow p$

Switch the hypothesis and conclusion

- Example:
  - **If we confess our sins, then he is faithful and just to forgive us our sins.**
    - p = we confess our sins
    - q = he is faithful and just to forgive us our sins
  - Converse = If he is faithful and just to forgive us our sins, then we confess our sins.
  - Does not necessarily make a true statement (It doesn't even make any sense.)

2.2 Analyze Conditional Statements

Negation

~p

Turn it to the opposite.

- Example:
  - The board is white.

ANS: → The board is not white.
ANS: → If it is not Wednesday, then there is rec.

2.2 Analyze Conditional Statements

Inverse

$\sim p \rightarrow \sim q$

Negating both the hypothesis and conclusion

- Example:
  - **If we confess our sins, then he is faithful and just to forgive us our sins.**
    - p = we confess our sins
    - q = he is faithful and just to forgive us our sins
  - Inverse = If we don't confess our sins, then he is not faithful and just to forgive us our sins.
  - Not necessarily true (He could forgive anyway)

2.2 Analyze Conditional Statements

Contrapositive

~q → ~p

Take the converse of the inverse

- Example:
  - If we confess our sins, then he is faithful and just to forgive us our sins.
    - p = we confess our sins
    - q = he is faithful and just to forgive us our sins
  - Contrapositive (inverse of converse) = If he is not faithful and just to forgive us our sins, then we won't confess our sins.
  - Always true.

ANS: → If there is rec, then it is not Wednesday.

2.2 Analyze Conditional Statements

- Write the following in If-Then form and then write the converse, inverse, and contrapositive
  - All whales are mammals.

If-Then: If it is a whale, then it is a mammal.
Converse: If it is a mammal, then it is a whale.
Inverse: If it is not a whale, then it is not a mammal.
Contrapositive: if it is not a mammal, then it is not a whale.

2.2 Analyze Conditional Statements

Biconditional Statement

- Logical statement where the if-then and converse are both true
- Written with “if and only if”
  - iff

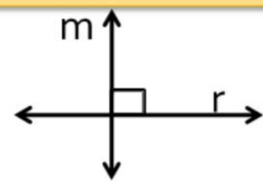
An angle is a right angle if and only if it measure 90°.

Red is hypothesis, Gray is conclusion

2.2 Analyze Conditional Statements

- All definitions can be written as if-then and biconditional statements

Perpendicular Lines
Lines that intersect to form right angles
$m \perp r$

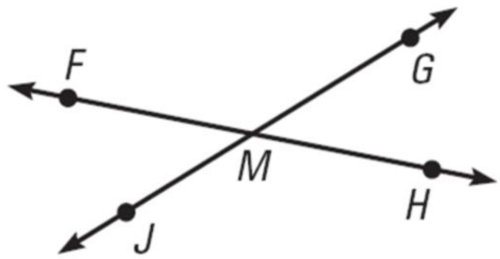


If-then: If lines intersect to form right angles, then they are perpendicular.
Biconditional: Lines are perpendicular iff they intersect to form right angles.

2.2 Analyze Conditional Statements

- Use the diagram shown. Decide whether each statement is true. *Explain* your answer using the definitions you have learned.

1. ∠JMF and ∠FMG are supplementary
2. Point M is the midpoint of $\overline{FH}$
3. ∠JMF and ∠HMG are vertical angles.
4. $\overleftrightarrow{FH} \perp \overleftrightarrow{JG}$



1. True, linear pairs are supplementary
2. False, no information given
3. True, intersecting lines form vertical angles
4. False, no information given

2.2 Analyze Conditional Statements

- *82 #4-20 even, 26, 28, 32, 36-52 even, 53-55 all = 24 total*

2.2 Answers and Quiz

- 2.2 Answers
- 2.2 Homework Quiz

2.3 Apply Deductive Reasoning

Deductive Reasoning

Use facts, definitions, properties, laws of logic to form an argument.

- Deductive reasoning
  - Always true
  - General → specific
- Inductive reasoning
  - Sometimes true
  - Specific → general

2.3 Apply Deductive Reasoning

Law of Detachment

If the hypothesis of a true conditional statement is true, then the conclusion is also true.

Detach means comes apart, so the 1st statement is taken apart.

- Example:
  1. **If we confess our sins, then He is faithful and just to forgive us our sins.** 1 John 1:9
  2. Jonny confesses his sins
  3. God is faithful and just to forgive Jonny his sins

2.3 Apply Deductive Reasoning

1. If you love me, keep my commandments.
2. I love God.
3. ____

1. If you love me, keep my commandments.
2. I keep all the commandments.
3. ____

I keep the commandments

Not Valid

2.3 Apply Deductive Reasoning

Law of Syllogism

If hypothesis p, then conclusion q.
If hypothesis q, then conclusion r.

If hypothesis p, then conclusion r.

If these statement are true,

then this statement is true

1. If we confess our sins, **He is faithful and just to forgive us our sins.**
2. **If He is faithful and just to forgive us our sins,** then we are blameless.
3. If we confess our sins, then we are blameless.

2.3 Apply Deductive Reasoning

- If you love me, keep my commandments.
- If you keep my commandments, you will be happy.
- ______________________________________

- If you love me, keep my commandments.
- If you love me, then you will pray.
- ______________________________________

- *90 #4-12 even, 16-28 even, 30-38 all = 20 total*
- *Extra Credit 93 #2, 4 = +2 total*

2.3 Answers and Quiz

- 2.3 Answers
- 2.3 Homework Quiz

2.4 Use Postulates and Diagrams

Postulates (axioms)

- Rules that are accepted without proof (assumed)

Theorem

- Rules that are accepted only with proof

2.4 Use Postulates and Diagrams

Basic Postulates (Memorize for quiz!)

- Through any two points there exists exactly one line.
- A line contains at least two points.
- If two lines intersect, then their intersection is exactly one point.
- Through any three noncollinear points there exists exactly one plane.

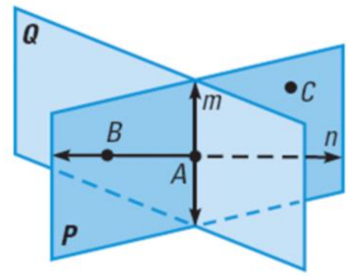
2.4 Use Postulates and Diagrams

Basic Postulates (continued)

- A plane contains at least three noncollinear points.
- If two points lie in a plane, then the line containing them lies in the plane.
- If two planes intersect, then their intersection is a line.

2.4 Use Postulates and Diagrams

- Which postulate allows you to say that the intersection of plane *P* and plane *Q* is a line?
- Use the diagram in Example 2 to write examples of Postulates 5, 6, and 7.

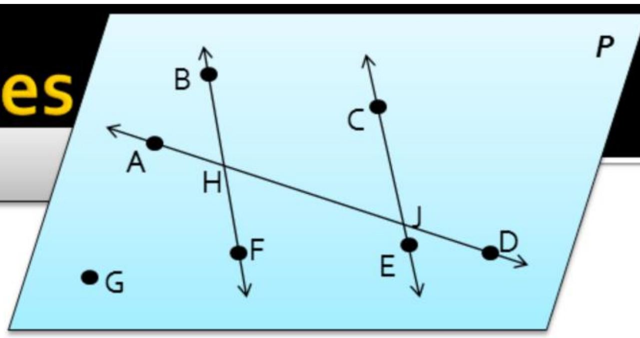


If two planes intersect, then their intersection is a line.

5: Line *n* passes through points A and B.
6: Line n contains points A and B
7: Line m and line n intersect at point A

2.4 Use Postulates

Interpreting a Diagram



You can Assume

- All points shown are coplanar
- ∠AHB and ∠BHD are a linear pair
- ∠AHF and ∠BHD are vertical angles
- A, H, J, and D are collinear
- $\overleftrightarrow{AD}$ and $\overleftrightarrow{BF}$ intersect at H

You cannot Assume

- G, F, and E are collinear
- $\overleftrightarrow{BF}$ and $\overleftrightarrow{CE}$ intersect
- $\overleftrightarrow{BF}$ and $\overleftrightarrow{CE}$ do not intersect
- ∠BHA ≅ ∠CJA
- $\overleftrightarrow{AD} \perp \overleftrightarrow{BF}$
- m∠AHB = 90°

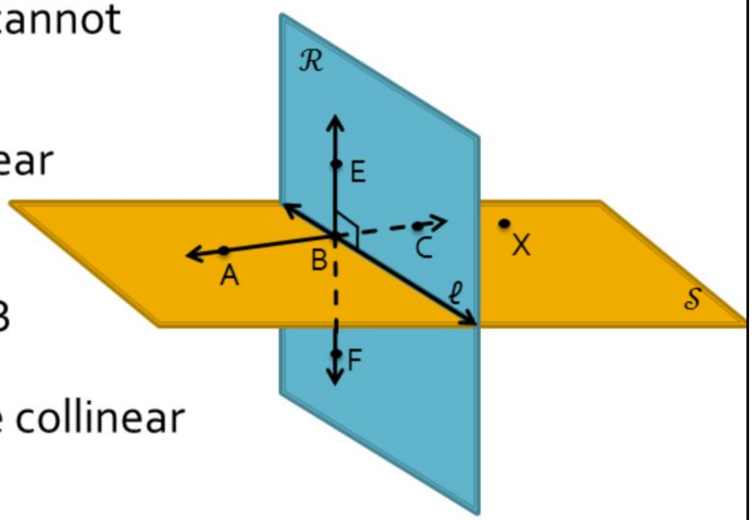
2.4 Use Postulates and Diagrams

- Sketch a diagram showing $\overrightarrow{FH} \perp \overline{EG}$ at its midpoint M.

2.4 Use Postulates and Diagrams

- Which of the follow cannot be assumed.

- A, B, and C are collinear
- $\overleftrightarrow{EF} \perp$ line ℓ
- $\overleftrightarrow{BC} \perp$ plane $\mathcal{R}$
- $\overleftrightarrow{EF}$ intersects $\overleftrightarrow{AC}$ at B
- line ℓ $\perp \overleftrightarrow{AB}$
- Points B, C, and X are collinear



BC ⊥ plane R
line ℓ ⊥ AB
Points B, C, and X are collinear

2.4 Use Postulates and Diagrams

- *99 #2-28 even, 34, 40-56 even = 24 total*

2.4 Answers and Quiz

- 2.4 Answers
- 2.4 Homework Quiz

2.5 Reasoning Using Properties from Algebra

- When you solve an algebra equation, you use properties of algebra to justify each step.
- Segment length and angle measure are real numbers just like variables, so you can solve equations from geometry using properties from algebra to justify each step.

2.5 Reasoning Using Properties

Property of Equality	Numbers	Segments	Angles
Reflexive	a = a	AB = AB	m∠1 = m∠1
Symmetric	a = b, then b = a	AB = CD, then CD = AB	m∠1 = m∠2, then m∠2 = m∠1
Transitive	a = b and b = c, then a = c	AB = BC and BC = CD, then AB = CD	m∠1 = m∠2 and m∠2 = m∠3, then m∠1 = m∠3
Add and Subtract	If a = b, then a+c= b+c	AB = BC, then AB + DE = BC + DE	m∠1 = m∠2, then m∠1 + m∠3 = m∠2 + m∠3
Multiply and divide	If a = b, then ac = bc	AB = BC, then 2AB = 2BC	m∠1 = m∠2, then 2m∠1 = 2m∠2
Substitution	If a = b, then a may be replaced by b in any equation or expression		
Distributive	a(b + c) = ab + ac		

2.5 Reasoning Using Properties from Algebra

- Name the property of equality the statement illustrates.
  - If $m\angle 6 = m\angle 7$, then $m\angle 7 = m\angle 6$.
  - If JK = KL and KL = 12, then JK = 12.
  - $m\angle W = m\angle W$

Symmetric
Transitive
Reflexive

2.5 Reasoning Using Properties from Algebra

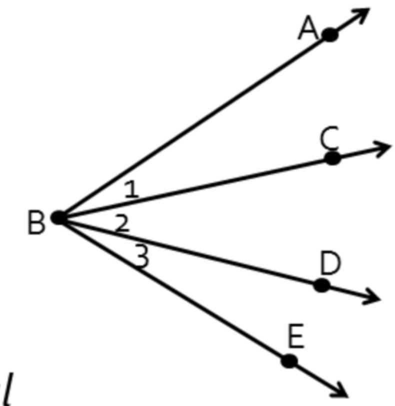
- Solve the equation and write a reason for each step
  - $14x + 3(7 - x) = -1$
- Solve $A = \frac{1}{2}bh$ for b.

$14x + 21 - 3x = -1$	distributive
$11x + 21 = -1$	definition of add (optional step)
$11x = -22$	subtraction
$x = -2$	division

$A = \frac{1}{2}bh$	
$2A = bh$	multiplication
$2A/h = b$	division
$b = 2A/h$	symmetric

2.5 Reasoning Using Properties from Algebra

- Given: m∠ABD = m∠CBE
- Show that m∠1 = m∠3

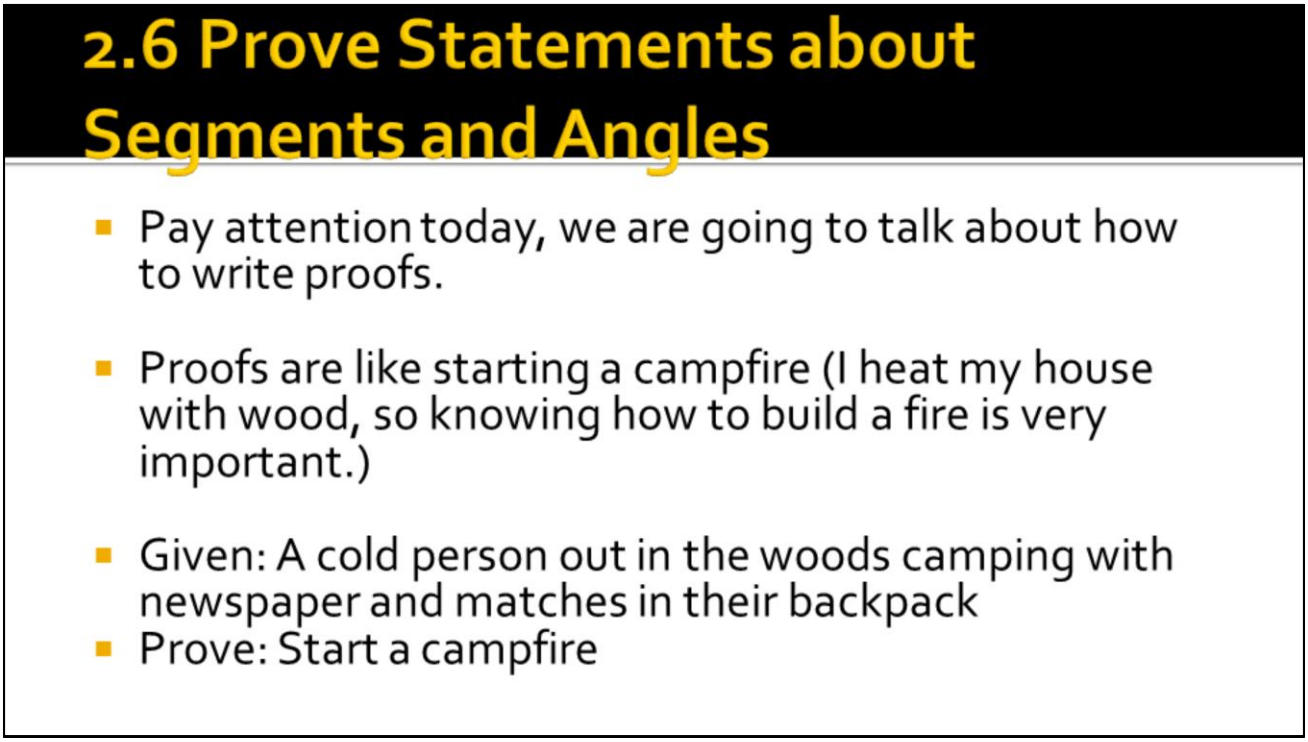


- *108 #4-34 even, 39-42 all = 20 total*
- *Extra Credit 111 #2, 4 = +2*

m∠ABD = m∠CBE	(given)
m∠ABD = m∠1 + m∠2	(angle addition post.)
m∠CBE = m∠2 + m∠3	(angle addition post.)
m∠1 + m∠2 = m∠2 + m∠3	(substitution)
m∠1 = m∠3	(subtraction)

2.5 Answers and Quiz

- 2.5 Answers
- 2.5 Homework Quiz



Ask: What do we do now? (write down ideas generated on the board)
Ask: Is there any special order for this? (yes, there is and have students start to put the steps in order)
If a student wants a step, such as put crumpled paper under wood, respond by asking where the paper came from. You cannot use an object until you get it.
Write the steps in the first column with justifications for each step in the second column

Cold person with newspaper and matches in their backpack (*given*)
Get dry wood from ground (*need something to burn*)
Break some wood into tender (*big pieces of wood don't readily start on fire*)
Put the rest of the wood in a pile near the fire location (*need something handy to burn*)
Get newspaper from backpack (*need something to start fire*)
Get matches from backpack (*need something to set fire*)
Clear area (*don't want to start forest fire*)
Crumple newspaper and put on ground (*newspaper is good for starting fires*)
Pile tender around the newspaper to make a "tepee". (*once the newspaper is started on fire, its heat will start the tender on fire*)
Strike matches (*matches have to be burning before it will start the paper*)
Use lit matches to start paper on fire in several places (*paper is the easiest thing to start on fire*)
Add bigger pieces of dry wood as the fire gets larger (*the tender will burn out quickly*)
You now have a campfire (*bigger pieces of wood are burning now and producing heat*)

2.6 Prove Statements about Segments and Angles

Congruence of segments and angles is reflexive, symmetric, and transitive.

- Writing proofs follow the same step as the fire.
  1. Write the given and prove written at the top for reference
  2. Start with the given as step 1
  3. The steps need to be in an logical order
  4. You cannot use an object without it being in the problem
  5. Remember the hypothesis states the object you are working with, the conclusion states what you are doing with it
  6. If you get stuck ask, "Okay, now I have ________. What do I know about ______?" and look at the hypotheses of your theorems, definitions, and properties.

2.6 Prove Statements about Segments and Angles

Complete the proof by justifying each

Given: Points P,Q, R, and S are collinear

Prove: PQ = PS – QS

P Q R S

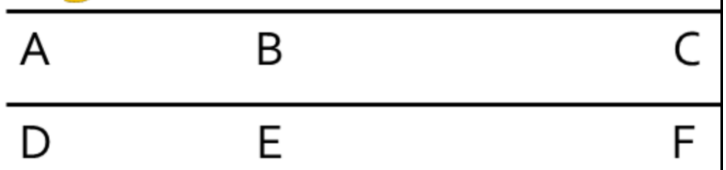
Statements	Reasons
Points P,Q, R, and S are collinear	*Given*
PS = PQ + QS	*Segment addition post*
PS – QS = PQ	*Subtraction*
PQ = PS – QS	*Symmetric*

Students are to come up with reasons

2.6 Prove Statements about Segments and Angles

Write a two column proof

Given: $\overline{AC} \cong \overline{DF}, \overline{AB} \cong \overline{DE}$
Prove: $\overline{BC} \cong \overline{EF}$



- *116 #2-12 even, 16, 18, 22-26 even, 30-36 all = 18 total*

AC ≅ DF, AB ≅ DE (given)
AC = DF, AB = DE (def ≅ segments)
AC – AB = DF – DE (subtraction =)
AC = AB + BC, DF = DE + EF (segment addition post)
AC – AB = BC, DF – DE = EF (subtraction =)
DF – DE = BC (substitution =)
BC = EF (substitution =)
BC ≅ EF (def ≅ segments)

2.6 Answers and Quiz

- 2.6 Answers
- 2.6 Homework Quiz

2.7 Prove Angle Pair Relationships

All right angles are congruent

Congruent Supplements Theorem

If two angles are supplementary to the same angle (or to congruent angles), then they are congruent

Congruent Complements Theorem

If two angles are complementary to the same angle (or to congruent angles), then they are congruent

2.7 Prove Angle Pair Relationships

Linear Pair Postulate

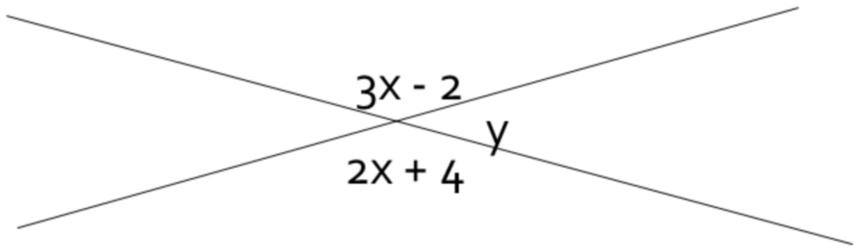
If two angles form a linear pair, then they are supplementary

Vertical Angles Congruence Theorem

Vertical angles are congruent

2.7 Prove Angle Pair Relationships

- Find x and y

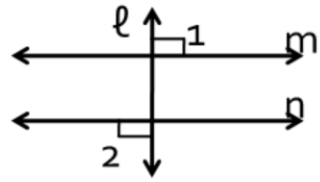


3x – 2 = 2x + 4 → x = 6

y = 180 – (3x – 2) = 180 – (3(6) + 4) = 180 – (18 + 4) = 180 – 22 = 158

2.7 Prove Angle Pair Relationships

- Given: ℓ ⊥ m, ℓ ⊥ n
- Prove: ∠1 ≅ ∠2

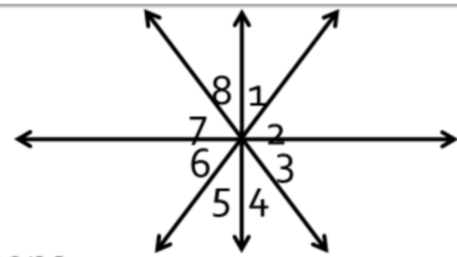


Statements	Reasons

ℓ ⊥ m, ℓ ⊥ n (given)
∠1 and ∠2 are right angles (def of ⊥ lines)
∠1 ≅ ∠2 (All rt ∠'s are ≅)

2.7 Prove Angle Pair Relationships

- Given:
  - $\angle 1$ and $\angle 3$ are complements
  - $\angle 3$ and $\angle 5$ are complements
- Prove: $\angle 1 \cong \angle 5$



Statements	Reasons

$\angle 1$ and $\angle 3$ are complements (given)
$\angle 3$ and $\angle 5$ are complement (given)
$\angle 1 \cong \angle 5$ (congruent complements theorem)

2.7 Prove Angle Pair Relationships

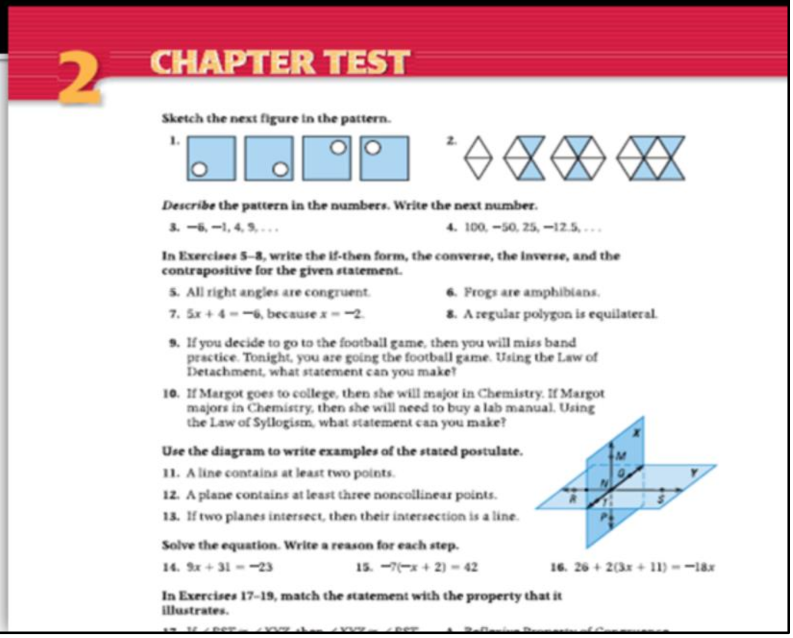
- *127 #2-28 even, 32-46 even, 50, 52 = 24 total*
- *Extra Credit 131 #2, 4 = +2*

2.7 Answers and Quiz

- 2.7 Answers
- 2.7 Homework Quiz

2.Review

- *138 #1-21 = 21 total*



2 CHAPTER TEST

Sketch the next figure in the pattern.

1.

2.

Describe **the pattern in the numbers. Write the next number.**

3. −6, −1, 4, 9, . . .
4. 100, −50, 25, −12.5, . . .

In Exercises 5–8, write the if-then form, the converse, the inverse, and the contrapositive for the given statement.

5. All right angles are congruent.
6. Frogs are amphibians.
7. $5x + 4 = -6$, because $x = -2$.
8. A regular polygon is equilateral.
9. If you decide to go to the football game, then you will miss band practice. Tonight, you are going the football game. Using the Law of Detachment, what statement can you make?
10. If Margot goes to college, then she will major in Chemistry. If Margot majors in Chemistry, then she will need to buy a lab manual. Using the Law of Syllogism, what statement can you make?

Use the diagram to write examples of the stated postulate.

11. A line contains at least two points.
12. A plane contains at least three noncollinear points.
13. If two planes intersect, then their intersection is a line.

Solve the equation. Write a reason for each step.

14. $9x + 31 = -23$
15. $-7(-x + 2) = 42$
16. $26 + 2(3x + 11) = -18x$

In Exercises 17–19, match the statement with the property that it illustrates.